1.1 Problem Solving: The Problem-Solving Process and Strategies

Objective:
1. Know Polya’s Four Steps to problem solving
2. Practice solving problems using problem solving and processes and strategies
3. See examples of solving problems that use guess and test, drawing a picture, and using variables strategies.

Problem Solving Strategies
1. Guess and Test
2. Draw a Picture
3. Use a Variable
4. Look for a Pattern
5. Make a List
6. Solve a Simpler Problem

Example 1
Place the whole numbers 1 through 9 in the circles in the triangle so that the sum of the numbers on each side is 17.

Polya’s Four Steps
Step 1: Understand the Problem
Step 2: Devise a Plan
Step 3: Carry Out the Plan
Step 4: Look Back

Example 2
Use any strategy you know to solve the next problem. As you solve this problem, pay close attention to the thought processes and steps that you use. Write down these strategies and compare them to a classmate’s. Are there similarities in your approaches to solving this problem?

Clues
- A phase similar to “for any number” is present or implied.
- A problem suggests an equation.
- A proof or a general solution is required.
- A problem contains phrases such as “consecutive,” “even,” or “odd” whole numbers.
- There is a large number of cases.
- There is an unknown quantity related to known quantities.
- There is an infinite number of numbers involved.
- You are trying to develop a general formula.

Method 1: Guess and Test
The Guess and Test strategy may be appropriate when
- There is a limited number of possible answers to test.
- You want to gain a better understanding of the problem.
- You have a good idea of what the answer is.
- You can systematically try possible answers.
- Your choices have been narrowed down by the use of other strategies.
- There is no other obvious strategy to try.

Method 2: Draw a Picture
The Draw a Picture strategy may be appropriate when
- A physical situation is involved.
- Geometric figures or measurements are involved.
- You want to gain a better understanding of the problem.
- A visual representation of the problem is possible.

Method 3: Use a Variable
The Use a Variable strategy may be appropriate when
- A phase similar to “for any number” is present or implied.
- A problem suggests an equation.
- A proof or a general solution is required.
- A problem contains phrases such as “consecutive,” “even,” or “odd” whole numbers.
- There is a large number of cases.
- There is an unknown quantity related to known quantities.
- There is an infinite number of numbers involved.
- You are trying to develop a general formula.

Group Assignment
Work in your groups to solve one of the problems on the activity sheet. Be prepared to share out your solution methods.

Strategy 1: Guess and Test (Check)
Strategy 2: Draw a Picture
Strategy 3: Use a Variable

When you finish your problem, try to solve it using another method.
Santa Ana College Math 204
Problem Solving Activity

Work in your groups to solve one of the problems below. Be prepared to share out your solution methods. When you finish your problem, try to solve it using another method.

**strategy 1** Guess and Test

**Problem**

Place the digits 1, 2, 3, 4, 5, 6 in the circles in Figure 1.2 so that the sum of the three numbers on each side of the triangle is 12.

![Figure 1.2](image_url)

**Additional Problems Where the Strategy “Guess and Test” Is Useful**

1.1 In the following cryptarithm—that is, a collection of words where the letters represent numbers—sun and four represent two three-digit numbers, and swim is their four-digit sum. Using all of the digits 0, 1, 2, 3, 6, 7, and 9 in place of the letters where no letter represents two different digits, determine the values of each letter.

sun
+ four
-----
swim

**strategy 2** Draw a Picture

**Problem**

Can you cut a pizza into 11 pieces with four straight cuts?

![Figure 1.10](image_url)

**Additional Problems Where the Strategy “Draw a Picture” Is Useful**

1.1 A tetromino is a shape made up of four squares where the squares must be joined along an entire side (Figure 1.10). How many different tetromino shapes are possible?

![Not a tetromino](image_url)
![A tetromino](image_url)

**strategy 3** Use a Variable

**Problem**

What is the greatest number that evenly divides the sum of any three consecutive whole numbers?

By trying several examples, you might guess that 3 is the greatest such number. However, it is necessary to use a variable to account for all possible instances of three consecutive numbers.

**Additional Problems Where the Strategy “Use a Variable” Is Useful**

1.1 Find the sum of the first 10, 100, and 500 counting numbers.
1.2 Problem Solving: Three Additional Strategies

Objective:
1. See examples of solving problems that look for patterns, make a list, and solving simpler problems.
2. Know the terms sequence, ellipsis, inductive reasoning
3. Identify various sequences of numbers: odd, even, square, powers of 3, and Fibonacci Sequence
4. Use inductive reasoning to predict the next terms in a given sequence

Problem Solving Strategies
- Guess and Test
- Make a List
- Use a Picture
- Make a Table
- Use a Variable
- Solve a Simpler Problem

Example 1
Without a calculator, find each sum.

\[
\begin{align*}
1 + 3 &= 4 \\
1 + 3 + 5 &= 9 \\
1 + 3 + 5 + 7 &= 16 \\
1 + 3 + 5 + 7 + 9 &= 25 \\
\ldots \\
1 + 3 + 5 + 7 + 9 + \ldots + 19 &= 100
\end{align*}
\]

Key Terms to Know
A pattern of numbers arranged in a particular order is called a number sequence, and the individual numbers in the sequence are called terms of the sequence. The counting numbers, 1, 2, 3, 4, give rise to many sequences. (An ellipsis, the three periods after the , means “and so on.”) Several sequences of counting numbers follow.

Inductive reasoning is used to draw conclusions or make predictions about a large collection of objects or numbers, based on a small representative subcollection.

Example 2
Find the ones digit in \(3^9\)

Strategy 5: Make a list
Problem
The number 10 can be expressed as the sum of four odd numbers in three ways: \(1+7+1+1,\quad 3+5+1+1,\text{ and }6+4+1+1\). How many ways can 10 be expressed as the sum of eight odd numbers?

Example 3
In a game where 3 darts are thrown, how many different scores are possible?

Strategy 4: Solve a simpler problem
Problem
In a group of nine coins, eight weigh the same and the ninth is heavier. Assume that the coins are identical in appearance. Using a pan balance, what is the smallest number of weighings needed to identify the heavier coin?

Example 4
How many different downward paths are there from Point A to Point B?
Homework 1.1: Section 1.1B: page 19-21: #2, 3, 20, 23 (don’t have to use website), 26, 32


3. Carol bought some items at a variety store. All the items were the same price, and she bought as many items as the price of each item in cents. (For example, if the items cost 10 cents, she would have bought 10 of them.) Her bill was $2.25. How many items did Carol buy?

20. An additive magic square has the same sum in each row, column, and diagonal. Find the error in this magic square and correct it.

| 47 | 56 | 34 | 22 | 83 | 7 |
| 24 | 67 | 44 | 26 | 13 | 75 |
| 29 | 52 | 3 | 99 | 18 | 48 |
| 17 | 49 | 89 | 4 | 53 | 37 |
| 97 | 6 | 3 | 11 | 74 | 28 |
| 35 | 19 | 46 | 87 | 8 | 54 |

23. Arrange the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 in the circles below so the sum of the numbers along each line of four is 20.

26. Solve and Explain

The equation \( \frac{y}{5} + 12 = 23 \) can be solved by subtracting 12
from both sides of the equation to yield \( \frac{y}{5} = 11 \) which can be
solved by multiplying both sides of the equation by 5 to obtain \( y = 55 \). Explain how this process is related to the Work Backward method described in Example 1.3.

32. ANALYZING STUDENT THINKING:

Consider the following problem:

The amount of fencing needed to enclose a rectangular field was 92 yards and the length of the field was 3 times as long as the width. What were the dimensions of the field?

Vance solved this problem by drawing a picture and using guess and test. Jolie set up an equation with \( x \) being the width of the field and solved it. They got the same answer but asked you which method was better. How would you respond?

Homework 1.2: Section 1.2B: page 34 - 37: #1, 3, 4, 10, 16, 24

1.3 The rectangular numbers are whole numbers that are represented by certain rectangular arrays of dots. The first five rectangular numbers are shown.

```
1   1   1   1   1
   2   3   4   5
   6   7   8   9
```

(a) Complete the following table and describe the pattern in the Number of Dots column.

<table>
<thead>
<tr>
<th>NUMBER</th>
<th>NUMBER OF DOTS (RECTANGULAR NUMBERS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

(b) Make a sketch to represent the seventh rectangular number.

(c) How many dots will be in the tenth rectangular number?

(d) Is there a rectangular number that has 180 dots in its shape? If so, which one?

(e) Write a formula for the number of dots in the \( n \)th rectangular number.

(f) What is the connection between triangular numbers (see Problem 1.6 in Set A) and rectangular numbers?

1.10

The Fibonacci sequence was defined to be the sequence 1, 1, 2, 3, 5, 8, 13, 21, ..., where each successive number is the sum of the preceding two. Observe the following pattern:

\[1+1=2, 2+3=5, 5+8=13, 8+13=21, \ldots\]

Write out six more terms of the Fibonacci sequence, and use the sequence to predict the answer to

\[1+1+2+3+5+\ldots=144 \]

without actually computing the sum. Then use your calculator to check your result.

1.4 The pentagonal numbers are whole numbers that are represented by pentagonal shapes. The first four pentagonal numbers are shown.
1.4 continued

(a) Complete the following table and describe the pattern in the Number of Dots column.

<table>
<thead>
<tr>
<th>NUMBER</th>
<th>NUMBER OF DOTS (PENTAGONAL NUMBERS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

(b) Make a sketch to represent the fifth pentagonal number.
(c) How many dots will be in the ninth pentagonal number?
(d) Is there a pentagonal number that has 200 dots in its shape? If so, which one?
(e) Write a formula for the number of dots in the nth pentagonal number.

1.16 A certain type of gutter comes in 6-foot, 8-foot, and 10-foot sections. How many different lengths can be formed using three sections of gutter?

Analyzing Student Thinking

1.24 Marietta extended the pattern 2, 4, 6 to be 2, 4, 8, 16, 32, ... Pascuel extended the same pattern to be 2, 4, 8, 14, 22, ... They asked you who was correct. How should you respond?